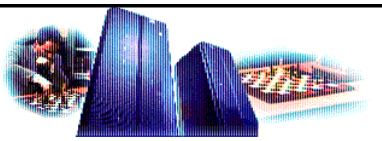

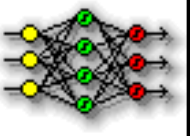

Sistemi Intelligenti
Reinforcement Learning: SARSA and Q-learning

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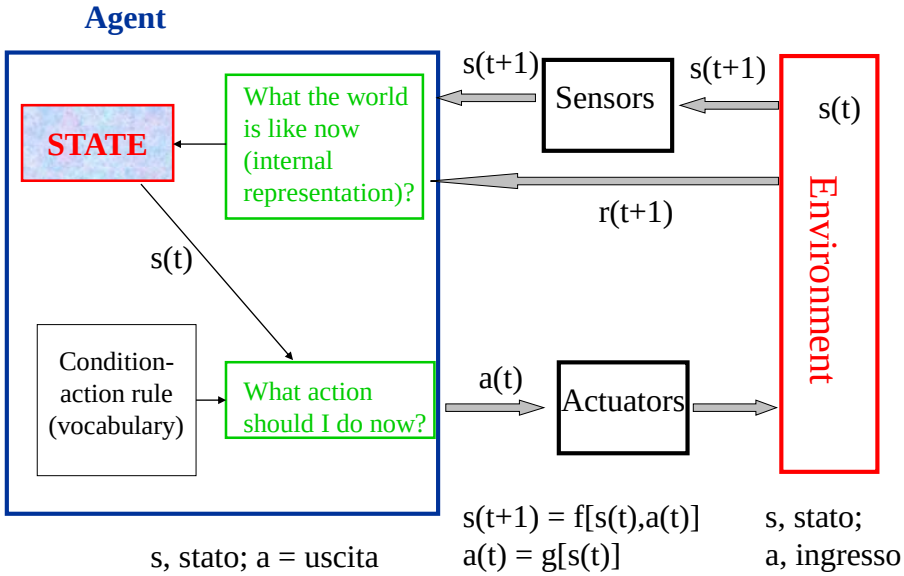
Schematic diagram of an agent

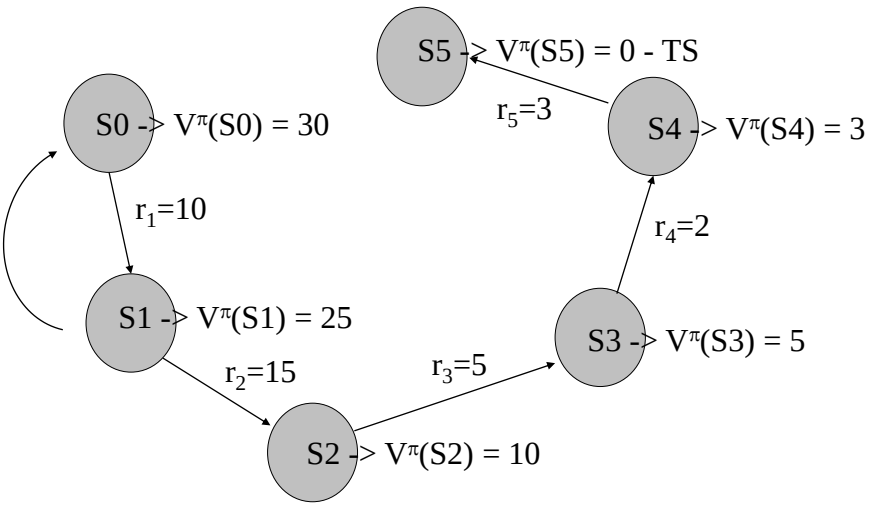


s, stato; a = uscita

s(t+1) = f[s(t),a(t)]
a(t) = g[s(t)]

s, stato;
a, ingresso

Learning $V_\pi(s)$



Come i diversi reward istantanei modificano $V^\pi(s)$?

Proprietà del metodo TD

Non richiede conoscenze a priori dell'ambiente.
L'agente stima dalle sue stesse stime precedenti (bootstrap).
Si dimostra che il metodo converge asintoticamente.

Batch vs trial learning.

Converge!!

$$V^{\pi}(s_t) = V^{\pi}(s_t) + \alpha\left[r_{t+1} + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t)\right]$$

Single backup, single state, s_t, single future state s_{t+1}

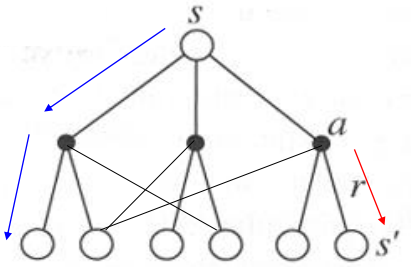
Rimpiazza iterative Policy evaluation.
Rimane il passo di Policy iteration (improvement).

TD(0)

Inizializziamo V(s) = 0.
Inizializziamo la policy: π(s,a).
Repeat
{ s = s0;
  Repeat // For each state until terminal state, analyze an episode
  { a = π(s);
    s_next = NextState(s, a);
    reward = Reward(s, s_next, a);
    V(s) = V(s) + α [reward + γ V(s next) – V(s)];
    s = s_next; }
  until TerminalState }
until convergence of V(s) for policy π(s,a)

Le value function

$$V^{\pi}(s)=E_{\pi}\{R_t \mid s_t=s\}=E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1}\,\middle|\, s_t=s\right\}=\left[\sum_{a_j}\pi(a_j,s)\right]\sum_{s'}P_{s\to s'|a_j}\left[R_{s\to s'|a_j}+\gamma V^{\pi}(s')\right]$$



$$Q^{\pi}(s,a)=E_{\pi}\{R_t \mid s_t=s,\ a_t=a\}=E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1}\,\middle|\, s_t=s, a_t=a\right\}$$

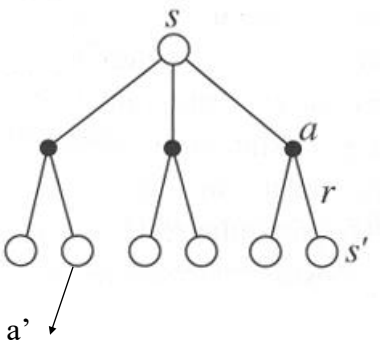
$$=\sum_{s'}P_{s\to s'|a_j}\left[R_{s\to s'|a_j}+\gamma V^{\pi}(s')\right]$$

Serve davvero la Value Function?

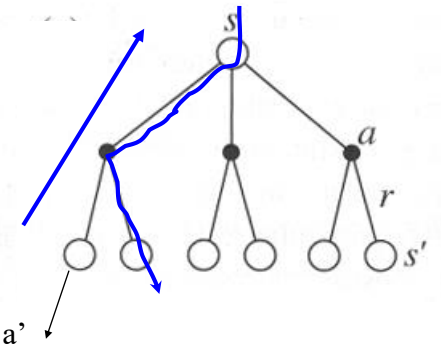
La Value Function deriva dalla visione della Programmazione Dinamica.

Ma è proprio necessario conoscere la Value function (che riguarda gli stati)? In fondo a noi interessa determinare la Policy.

Sample backup

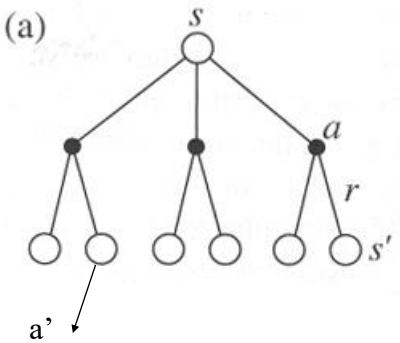


Full backup



Single sample is evaluated

Sample backup

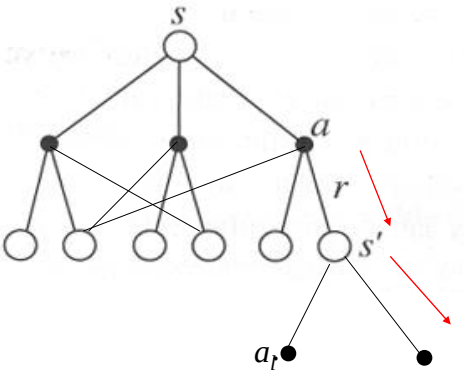


SARSA algorithm

State –
Action –
Reward –
State (next) –
Action (next)

Campionamento di s_{t+1} e di r_{t+1}

Calcolo ricorsivo della value function Q



$$Q^{\pi}(s,a) = E_{\pi}\{R_t \mid s_t = s,\ a_t = a\} = E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1} \,\middle|\, s_t = s, a_t = a\right\}$$

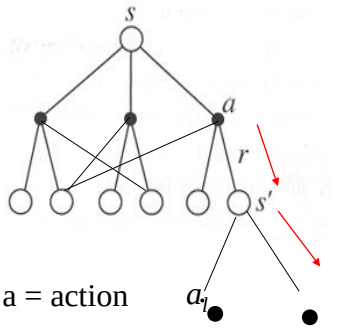
$$= \sum_{s'} P_{s\to s'|a}\left[R_{s\to s'|a} + \gamma \sum_{l}\pi(s', a_l)Q^{\pi}(s', a_l)\right]$$

Come apprendere Q: SARSA

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha\left[r_{t+1} + \gamma Q\left(s_{t+1}, a_{t+1}\right) - Q\left(s_t, a_t\right)\right]$$

1) Apprendiamo il valore di Q per una policy data (***on-policy***).

2) Dopo avere appreso la funzione Q, possiamo modificare la policy in modo da migliorarla (**policy improvement**)



s = state, a = action, r = reward, s = state, a = action è di tipo TD(0)

SARSA Algorithm (progetto)

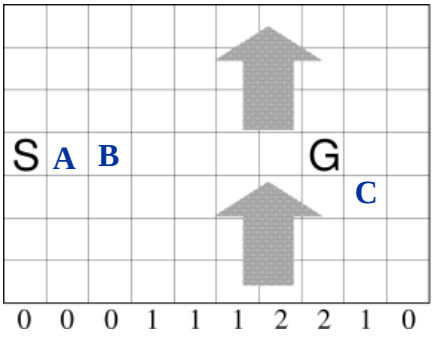
```
Q(s,a) = rand();    // ∀s, ∀a, eventualmente Q(s,a) = 0
Repeat                          // for each episode
{           s = s0;
            Repeat              // for each step of the single episode
            {
                        a = Policy(s);          // ε-greedy??
                        s_next = NextState(s,a);
                        reward = Reward(s,s_next,a);
                        a_next = Policy(s_next);        // ε-greedy?
                        Q(s,a) = Q(s,a) + α [reward + γ Q(s_next, a_next) – Q(s,a)];
                        s = s_next;
            }                   // until last state
}                               // until the end of learning
```

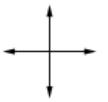
1) Apprendiamo il valore di Q per una policy data (on-policy).
2) Dopo avere appreso la funzione Q, possiamo modificare la policy in modo da migliorarla.

Come integrare i due passi?

Esempio

From Start to Goal.



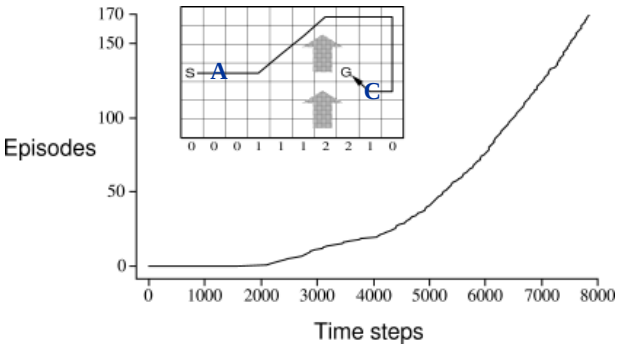


standard moves

Upwards wind

Q(s,a) iniziale = 0.
r = 0 se s' = G; altrimenti r = -1.
π(s,a) data.

Esempio - risultato



Policy π, greedy or ε-greedy

ε = 0.1
α= 0.5
γ = 1

Per trial or per epoch

Al termine, policy improvement.

Correzione di Q ad un passo:
Q(S, east) = 0 + 0.5 [-1 + 0 – 0] = -0.5
Q(A, east) = 0 + 0.5 [-1 + 0 –0] = -0.5
Q(C, west) = 0 + 0.5 [0 + 0 – 0] = 0; (NB c'è il vento verso l'alto di 1)

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha\left[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)\right]$$

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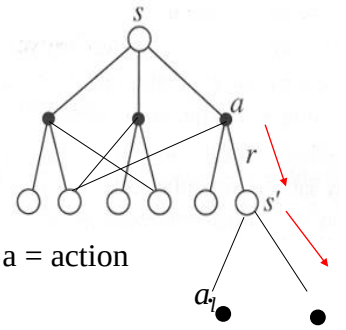
Come apprendere Q: SARSA

$$Q^{\pi}(s_t, a_t) = Q^{\pi}(s_t, a_t) + \alpha\left[r_{t+1} + \gamma Q^{\pi}(s_{t+1}, a_{t+1}) - Q^{\pi}(s_t, a_t)\right]$$

1) Apprendiamo il valore di Q per una policy data, π, (on-policy).

2) Dopo avere appreso la funzione Q, possiamo modificare la policy in modo da migliorarla (**policy improvement**)



S = state, a = action, r = reward, s = state, a = action

On-policy learning.

Value iteration

$$Q^{\pi}{}_{k+1}(s,a) = \sum_{s'} P_{s\to s'|a_j}\left\{R_{s\to s'|a_j} + \gamma\left[\sum_{a'_j}\pi(a_j, s')\right]Q^{\pi}{}_{k}(s', a'_j)\right\}$$

Invece di considerare una policy stocastica, consideriamo l'azione migliore in base al reward atteso a lungo termine per quella azione a':

$$Q_{k+1}(s,a) = \sum_{s'} P_{s\to s'|a}\left[R_{s\to s'|a} + \gamma \max_{a'} Q_k(s', a')\right]$$

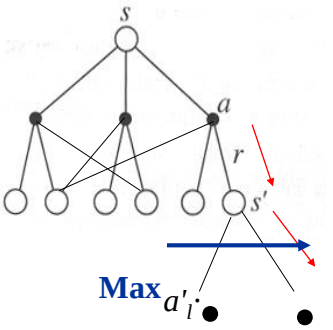
∀s

Off-policy Temporal Difference: Q-learning

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha\left[r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1},a_{t+1}) - Q(s_t,a_t) \right]$$

Non imparo semplicemente la funzione valore Q, ma la funzione valore Q ottima.

In s, scelgo un ramo del grafo, e poi **decido** ad un passo come continuare, guardando il reward a lungo termine stimato per le diverse azioni.

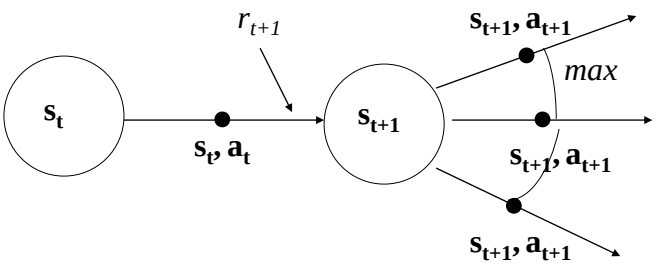


Q-learning algorithm (progetto)

```
Q(s,a) = 0;            // ∀s, ∀a,
Policy data
Repeat                                          // for each episode
{       s = s0; PolicyStable = TRUE;
        Repeat                                  // for each step of the single episode
        {       a = Policy(s);                  // eventualmente ε-greedy
                s_next = NextState(s,a);
                reward = Reward(s, s_next, a);
                a_next_pol = PolicyGreedy(s_next);          // on policy
                a_next = argmax(Q(s_next, a);
                                a

                if (a_next_pol != a_next)
                {   UpdatePolicy(s_next, a_next); PolicyStable = FALSE; }
                endif;
                Q(s,a) = Q(s,a) + α [reward + γ Q(s_next, a_next) – Q(s,a)];
                s = s_next;
                a = a_next;             // a = Policy(s = s_next)
        }   // until last state
}   // until the end of learning (PoicyStable = TRUE)
```

Rappresentazione grafica



$Q(s_t, a_t)$ $Q(s_{t+1}, a_{t+1})$

One step for **Q Iteration**

Viene migliorata la policy al tempo t+1.

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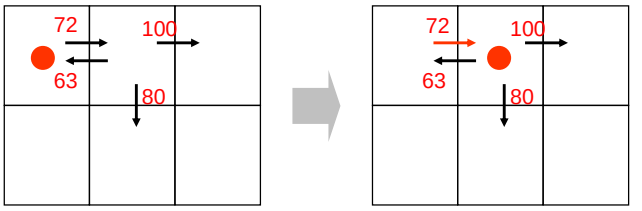
Example 1 – Q Learning Update

$\gamma = 0.9$



0 reward received in the transition. Q(.,.) initialized ≠ 0

Esempio tratto dai lucidi del corso di Brian C. Williams su RL.
Modificati dalle slide di: Manuela Veloso, Reid Simmons, & Tom Mitchell, CMU

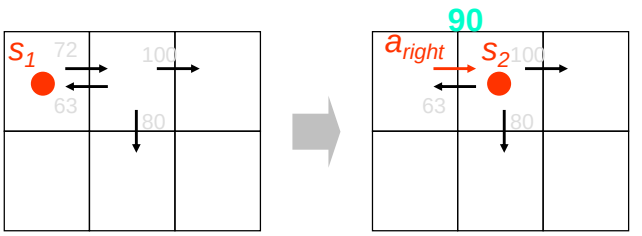
Apprendimento della funzione valore Q. Versione Q-learning. Q(A,dx) = ?

S_1	S_2	S_3
S_6	S_5	S_4

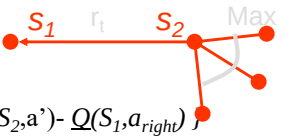
In rosso i valori di Q(s,a).
Nessun reward istantaneo.

Example 1 – Q Learning Update

$\gamma = 0.9$
$\alpha = 0.1$
$a(S_2)$ = down



0 reward received in the transition



$$\underline{Q}(S_1,a_{right}) \leftarrow \underline{Q}(S_1,a_{right}) + \alpha\{ r(A,a_{right}, S_2) + \gamma \max_{a'} \underline{Q}(S_2,a') - \underline{Q}(S_1,a_{right}) \}$$
$$\leftarrow 72 + \alpha [0 + 0.9 \max \{63, 80, 100\} - \underline{Q}(S_1,a_{right})]$$
$$\leftarrow 72 + \alpha(90 - 72) = 72 + 1.8 = 73.8$$

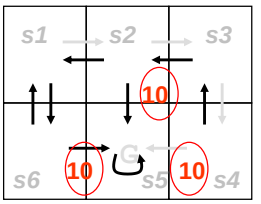
Correzione di $\underline{Q}(S_1,a_{right})$
Correzione dell'azione in S_2 da down a right
La correzione di $\underline{Q}(S_1,a_{right})$ va a 0
quando $\underline{Q}(S_1,a_{right}) = 90$

$Q(S_2,a_{down}) = 80$
$Q(S_2,a_{right}) = 100$
$Q(S_2,a_{left}) = 63$

Example 2: Q-Learning Iterations: Episodic

- Start at upper left; Initial selected policy: move clockwise; Q(s,a) initially 0; γ = 0.8. **Reward solo nelle transizioni**.

Reward istanteo in rosso e cerchiato



$\alpha = 1$

$$Q(s_t, a_t) \leftarrow \left[r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1}) \right]$$

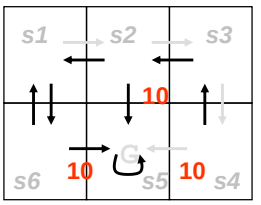
E.g. videogioco.
In G rimango in G - loop

Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0			

Q-Learning Iterations

- Start at upper left – move clockwise; table initially 0; γ = 0.8; α = 1

$$\underline{Q}(s, a) \leftarrow r + \gamma \max_{a'} \underline{Q}(s', a')$$

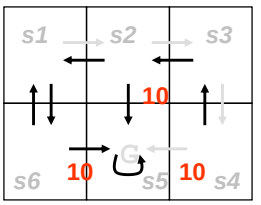


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$\underline{Q}(s, a) \leftarrow r + \gamma \max_{a'} \underline{Q}(s', a')$$

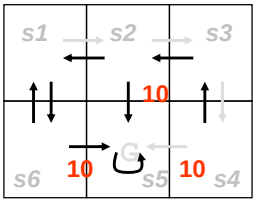


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{\underline{Q}(s5,a)$ = 10 + 0.8 x 0 = **10**

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$\underline{Q}(s, a) \leftarrow r + \gamma \max_{a'} \underline{Q}(s', a')$$

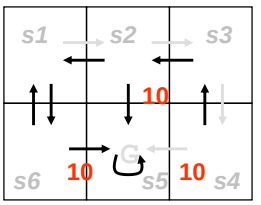


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \underline{Q}(s5,loop)$ = 10 + 0.8 x 0 = **10**
0	0	$r + \gamma \max_{a'} \{\underline{Q}(s4,W), \underline{Q}(s4,N)\}$ = 0 + 0.8 x max{10,0) = **8**	

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$\underline{Q}(s, a) \leftarrow r + \gamma \max_a \underline{Q}(s', a')$$

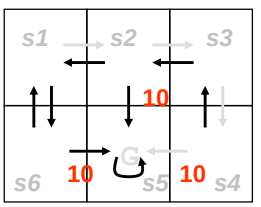


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \{\underline{Q}(s5,loop) - Q(s4,W)\} =$ $10 + 0.8 \times 0 - 0 =$ **10**
0	0	$r + \gamma \underline{Q}(s4,W)$ $= 0 + 0.8 \times 10 =$ **8**	10
0	$r + \gamma \max_{a'} \{\underline{Q}(s3,W), \underline{Q}(s3,S)\}$ $= 0 + 0.8 \times \max\{0,8\} =$ **6.4**		

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$\underline{Q}(s, a) \leftarrow r + \gamma \max_a \underline{Q}(s', a')$$

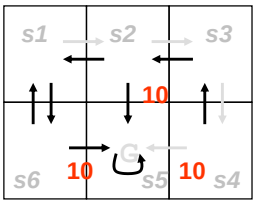


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \{\underline{Q}(s5,loop) - Q(s4,W)\} =$ $10 + 0.8 \times 0 - 0 =$ **10**
0	0	$r + \gamma \underline{Q}(s4,W)$ $= 0 + 0.8 \times 10 =$ **8**	10
0	$r + \gamma \max_{a'} \{\underline{Q}(s3,W), \underline{Q}(s3,S)\}$ $= 0 + 0.8 \times \max\{0,8\} =$ **6.4**		
$r + \gamma \max_{a'} \{\underline{Q}(s2,W), \underline{Q}(s2,S)\}$ $= 0 + 0.8 \times \max\{6.4, 0\} =$ **5.12**			

Q-Learning Iterations: improving policy

- Start at upper left – move clockwise; $\gamma = 0.8$; $\alpha = 1$

$$\underline{Q}(s, a) \leftarrow r + \gamma \max_{a'} \underline{Q}(s', a')$$



Mossa ε-greedy in s_2 (invece che a = E, scelto a = S):
calcolo $Q(s_2, S) = r + \gamma \max_{a'} \{\underline{Q}(s5,a')\} = 10 + 0.8 \times 0 =$ **10**

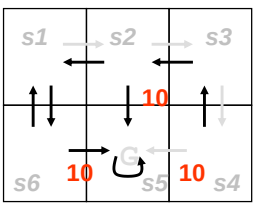
Episodio successivo:
Ricalcolo $Q(s_1, E) = r + \gamma \max_{a'} \{\underline{Q}(s2,E), Q(s2,W), Q(s2, S)\} =$
$r + \gamma \max_{a'} \{6.4, 0.0, 10.0\}$ ➔ **South = $\pi(s_2)$! Policy changed**

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

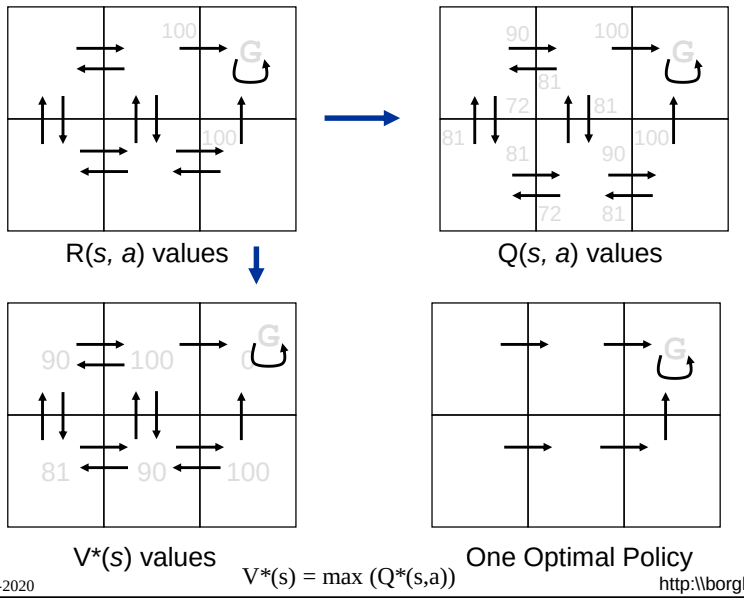
$$\underline{Q}(s, a) \leftarrow r + \gamma \max_{a'} \underline{Q}(s', a')$$

NB in s_2 the new policy drives the agent towards the s_5 state (loop).



Q(s1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{\underline{Q}(s5,loop)\} = 10 + 0.8 \times 0 = 10$
0	0	$r + \gamma \max_{a'} \{\underline{Q}(s4,W), \underline{Q}(s4,N)\} = 0 + 0.8 \times \max\{10,0) = 8$	10
0	$r + \gamma \max_{a'} \{\underline{Q}(s3,W), \underline{Q}(s3,S)\} = 0 + 0.8 \times \max\{0,8) = 6.4$	8	10
8	6.4	8	10

Example 3 - Comparison of functions V and Q (γ = 0.9)



V*(s) = max (Q*(s,a))

Proprietà del rinforzo

L'ambiente o l'interazione può essere complessa.

Il rinforzo può avvenire solo dopo una più o meno lunga sequenza di azioni (delayed reward).

E.g. agente = giocatore di scacchi.
ambiente = avversario.

Problemi collegati:
temporal credit assignement.
structural credit assignement.

L'apprendimento non è più da esempi, ma dall'osservazione del proprio comportamento nell'ambiente.

Esempio SW

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