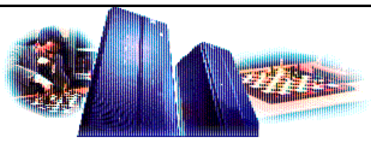

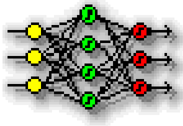

Sistemi Intelligenti Reinforcement Learning: Q-learning

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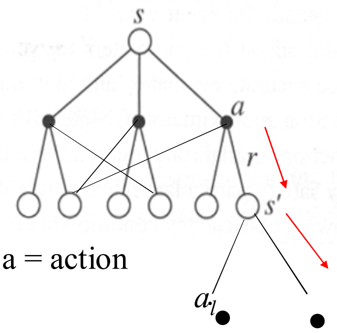
Come apprendere Q: SARSA

$$Q^{\pi}(s_t,a_t) = Q^{\pi}(s_t,a_t) + \alpha\left[r_{t+1} + \gamma Q^{\pi}(s_{t+1},a_{t+1}) - Q^{\pi}(s_t,a_t)\right]$$

1) Apprendiamo il valore di Q per una policy data, π, (on-policy).

2) Dopo avere appreso la funzione Q, possiamo modificare la policy in modo da migliorarla (**policy improvement**)



S = state, a = action, r = reward, s = state, a = action

On-policy learning.

Value iteration

$$Q^{\pi}_{k+1}(s,a) = \sum_{s'} P_{s \to s'|a_j} \left\{ R_{s \to s'|a_j} + \gamma \left[\sum_{a'_j} \pi(a_j, s') \right] Q^{\pi}_k(s', a'_j) \right\}$$

Invece di considerare una policy stocastica, consideriamo l'azione migliore in base al reward atteso a lungo termine per quella azione a':

$$Q_{k+1}(s,a) = \sum_{s'} P_{s \to s'|a} \left[R_{s \to s'|a} + \gamma \max_{a'} Q_k(s',a') \right]$$

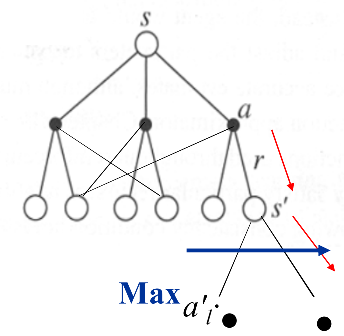
∀s

Off-policy Temporal Difference: Q-learning

$$\boxed{Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha\left[r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1},a_{t+1}) - Q(s_t,a_t)\right]}$$

Non imparo semplicemente la funzione valore Q, ma la funzione valore Q ottima.

In s, scelgo un ramo del grafo, e poi **decido** ad un passo come continuare, guardando il reward a lungo termine stimato per le diverse azioni.

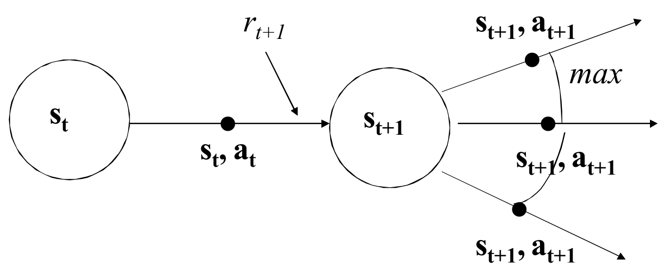


Q-learning algorithm (progetto)

```
Q(s,a) = 0;          // ∀s, ∀a,
Policy data
Repeat                                          // for each episode
{       s = s0; PolicyStable = TRUE;
        Repeat                                  // for each step of the single episode
        {       a = Policy(s);                  // eventualmente ε-greedy
                s_next = NextState(s,a);
                reward = Reward(s, s_next, a);
                a_next_pol = PolicyGreedy(s_next);          // on policy
                a_next = argmax(Q(s_next, a);
                             a

                if (a_next_pol != a_next)
                {    UpdatePolicy(s_next, a_next); PolicyStable = FALSE; }
                endif;
                Q(s,a) = Q(s,a) + α [reward + γ Q(s_next, a_next) – Q(s,a)];
                s = s_next;
                a = a_next;           // a = Policy(s = s_next)
        }    // until last state
}   // until the end of learning (PoicyStable = TRUE)
```

Rappresentazione grafica



$Q(s_t, a_t)$ $Q(s_{t+1}, a_{t+1})$

One step for **Q Iteration**

Viene migliorata la policy al tempo t+1.

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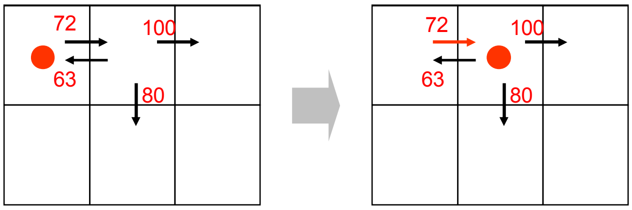
Example 1 – *Q* Learning Update

$\gamma = 0.9$



0 reward received in the transition. Q(.,.) initialized $\neq$ 0

Esempio tratto dai lucidi del corso di Brian C. Williams su RL.
Modificati dalle slide di: Manuela Veloso, Reid Simmons, & Tom Mitchell, CMU

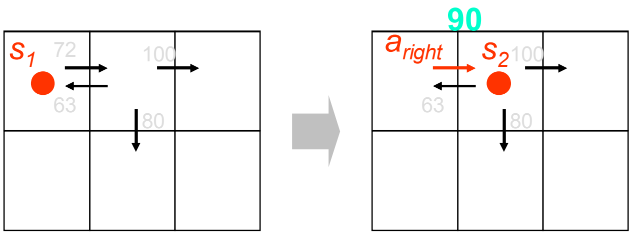
Apprendimento della funzione valore Q. Versione Q-learning. Q(A,dx) = ?

S_1	S_2	S_3
S_6	S_5	S_4

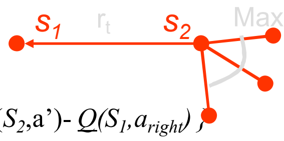
In rosso i valori di Q(s,a).
Nessun reward istantaneo.

Example 1 – *Q* Learning Update

$\gamma = 0.9$
$\alpha = 0.1$
$a(S_2)$ = down



0 reward received in the transition



$$Q(S_1,a_{right}) \leftarrow Q(S_1,a_{right}) + \alpha\{ r(A,a_{right}, S_2) + \gamma \max_{a'} Q(S_2,a') - Q(S_1,a_{right}) \}$$
$$\leftarrow 72 + \alpha\,[0 + 0.9 \max \{63, 80, 100\} - Q(S_1,a_{right})\,]$$
$$\leftarrow 72 + \alpha(90 - 72) = 72 + 1.8 = 73.8$$

Correzione di $Q(S_1,a_{right})$
Correzione dell'azione in S_2 da down a right
La correzione di $Q(S_1,a_{right})$ va a 0
quando $Q(S_1,a_{right}) = 90$

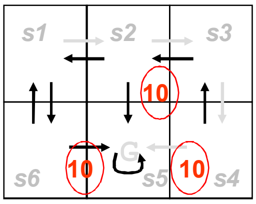
$Q(S_2,a_{down}) = 80$
$Q(S_2,a_{right}) = 100$
$Q(S_2,a_{left}) = 63$

Example 2: Q-Learning Iterations: Episodic

- Start at upper left; Initial selected policy: move clockwise; Q(s,a) initially 0; $\gamma = 0.8$. **Reward solo nelle transizioni**.

$\alpha = 1$

Reward istanteo in rosso e cerchiato



$$Q(s_t, a_t) \leftarrow \left[r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1}) \right]$$

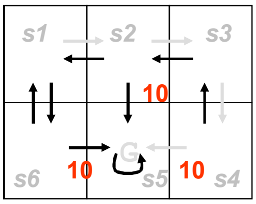
E.g. videogioco.
In G rimango in G - loop

Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0			

Q-Learning Iterations

- Start at upper left – move clockwise; table initially 0; $\gamma = 0.8$; $\alpha = 1$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s', a')$$

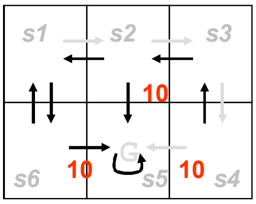


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

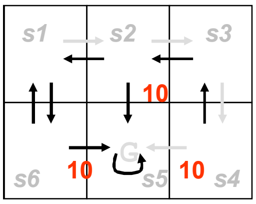


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{Q(s5,a) =$ $10 + 0.8 \times 0 = \mathbf{10}$

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

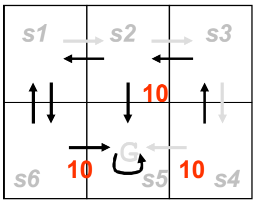


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma\, Q(s5,loop) =$ $10 + 0.8 \times 0 = \mathbf{10}$
0	0	$r + \gamma \max_{a'} \{Q(s4,W), Q(s4,N)\} =$ $0 + 0.8 \times \max\{10,0) = \mathbf{8}$	

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

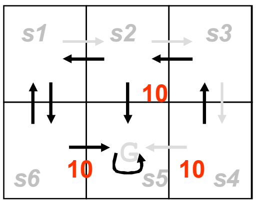


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma$ {Q(s5,loop)-Q(s4,W)}= 10 + 0.8 x 0 – 0 = **10**
0	0	$r + \gamma$ Q(s4,W) = 0 + 0.8 x 10 = **8**	10
0	$r + \gamma \max_{a'}$ {Q(s3,W), Q(s3,S)} = 0 + 0.8 x max{0,8} = **6.4**		

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

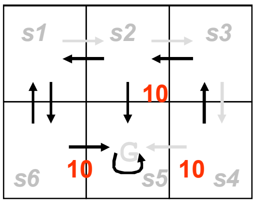


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma$ {Q(s5,loop)-Q(s4,W)}= 10 + 0.8 x 0 – 0 = **10**
0	0	$r + \gamma$ Q(s4,W) = 0 + 0.8 x 10 = **8**	10
0	$r + \gamma \max_{a'}$ {Q(s3,W), Q(s3,S)} = 0 + 0.8 x max{0,8} = **6.4**		
$r + \gamma \max_{a'}$ {Q(s2,W), Q(s2,S)} = 0 + 0.8 x max{6.4, 0} = **5.12**			

Q-Learning Iterations: improving policy

- Start at upper left – move clockwise; $\gamma = 0.8$; $\alpha = 1$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$



Mossa ε-greedy in s_2 (invece che a = E, scelto a = S):
calcolo $Q(s_2, S) = r + \gamma \max_{a'} \{Q(s5,a')\} = 10 + 0.8 \times 0 =$ **10**

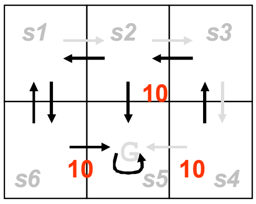
Episodio successivo:
Ricalcolo $Q(s_1, E) = r + \gamma \max_{a'} \{Q(s2,E), Q(s2,W), Q(s2, S)\} =$
$r + \gamma \max_{a'} \{6.4, 0.0, 10.0\}$ ➔ **South = $\pi(s_2)$! Policy changed**

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

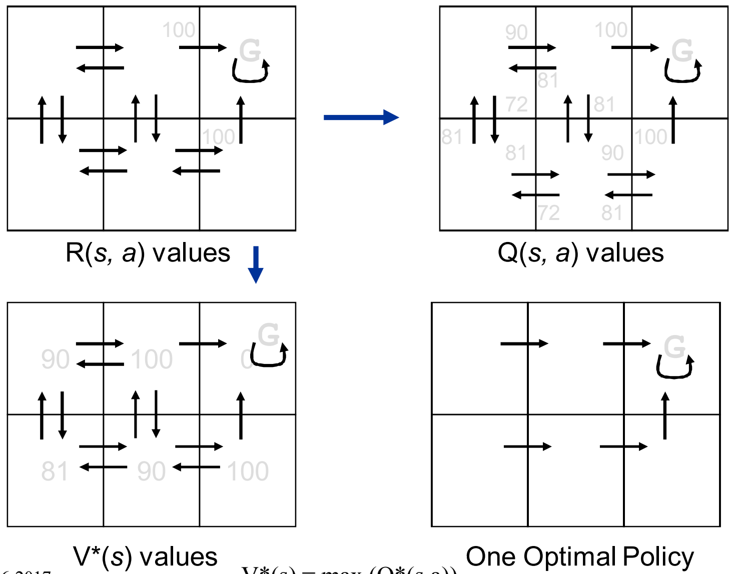
$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

NB in s_2 the new policy drives the agent towards the s_5 state (loop).



Q(s1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{Q(s5,loop)\} = 10 + 0.8 \times 0 = 10$
0	0	$r + \gamma \max_{a'} \{Q(s4,W), Q(s4,N)\} = 0 + 0.8 \times \max\{10,0\} = 8$	10
0	$r + \gamma \max_{a'} \{Q(s3,W), Q(s3,S)\} = 0 + 0.8 \times \max\{0,8\} = 6.4$	8	10
8	6.4	8	10

Example 3 - Comparison of functions V and Q (γ = 0.9)



$V^*(s) = \max\,(Q^*(s,a))$

Proprietà del rinforzo

L'ambiente o l'interazione può essere complessa.

Il rinforzo può avvenire solo dopo una più o meno lunga sequenza di azioni (delayed reward).

E.g. agente = giocatore di scacchi.
ambiente = avversario.

Problemi collegati:
- temporal credit assignement.
- structural credit assignement.

L'apprendimento non è più da esempi, ma dall'osservazione del proprio comportamento nell'ambiente.

Esempio SW

- Labirinto

- Gatto & Topo

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