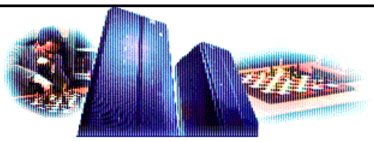

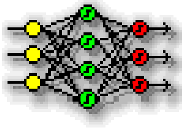

Sistemi Intelligenti Reinforcement Learning: Q-learning

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1° annual RL competition @ NIPS

http://rlai.cs.ualberta.ca/RLAI/rlc.html

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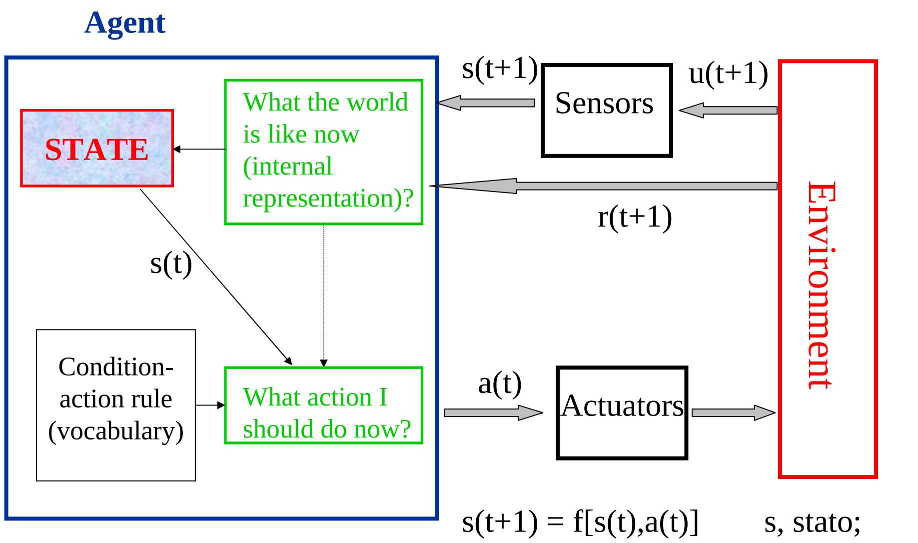
Schematic diagram of an agent



$s(t+1) = f[s(t),a(t)]$ | s, stato;
$\pi(s,a)$ è la policy | a, ingresso

s, stato; a, uscita

How About Learning the Value Function?

Facciamo imparare all'agente la value function, per una certa politica: V^π:

$$\boxed{V^\pi(s) = \left[\sum_{a_j} \pi(a_j, s)\right] \sum_{s'} P_{s \to s'|a_j} \left[R_{s \to s'|a_j} + \gamma V^\pi(s')\right]}$$

È una funzione dello stato.

Una volta imparata la value function, V^*, l'agente seleziona la policy ottima passo per passo, "one step lookahead":

$$\pi^*(s) = \arg\max_a \sum_{s'} P^a_{s->s'} \left[R^a_{s->s'} + \gamma V^\pi(s')\right]$$

Full backup, for all states

Value function iteration

Facciamo imparare all'agente la value function, per una certa politica: V^π, analizzando quello che succede in uno step temporale:

$$V^\pi{}_{k+1}(s) = \left[\sum_{a_j} \pi(a_j, s)\right] \sum_{s'} P_{s \to s'|a_j} \left[R_{s \to s'|a_j} + \gamma V^\pi{}_k(s')\right]$$

L'apprendimento della policy si può inglobare nella value iteration:

$$V_{k+1}(s) = \max_a \sum_{s'} P_{s \to s'|a} \left[R_{s \to s'|a} + \gamma V_k(s')\right]$$

Full backup, for all states

Asynchronous DP

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} P_{s\rightarrow s'|a}\left[R_{s\rightarrow s'|a} + \gamma V_k(s')\right]$$

Full backup, single state, s, all future states s'

Fino a questo punto, è noto un modello dell'ambiente:
- R(.)
- P(.)

Temporal Differences

Viene rimossa ogni conoscenza a-priori sull'ambiente.

Value iteration and policy iteration are merged into one update equation:

$$\boxed{V^\pi(s_t) = V^\pi(s_t) + \alpha\left[r_{t+1} + \gamma V^\pi(s_{t+1}) - V^\pi(s_t)\right]}$$

Single backup, single state, s_t, single future state s_{t+1}

+ policy improvement

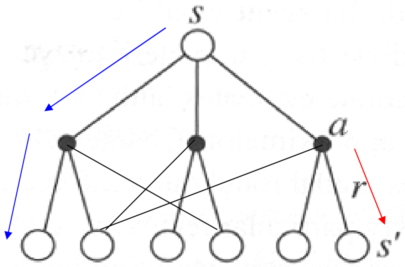
Serve davvero la Value Function?

La Value Function deriva dalla visione della Programmazione Dinamica.

Ma è proprio necessario conoscere la Value function? In fondo a noi interessa determinare la Policy.

Le value function

$$V^{\pi}(s)=E_{\pi}\{R_t \mid s_t=s\}=E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1} \middle| s_t=s\right\}=\left[\sum_{a_j}\pi(a_j,s)\right]\sum_{s'}P_{s\to s'|a_j}\left[R_{s\to s'|a_j}+\gamma V^{\pi}(s')\right]$$



$$Q^{\pi}(s,a)=E_{\pi}\{R_t \mid s_t=s,\ a_t=a\}=E_{\pi}\left\{\sum_{k=0}^{\infty}\gamma^k r_{t+k+1} \middle| s_t=s, a_t=a\right\}$$

$$=\sum_{s'}P_{s\to s'|a_j}\left[R_{s\to s'|a_j}+\gamma V^{\pi}(s')\right]$$

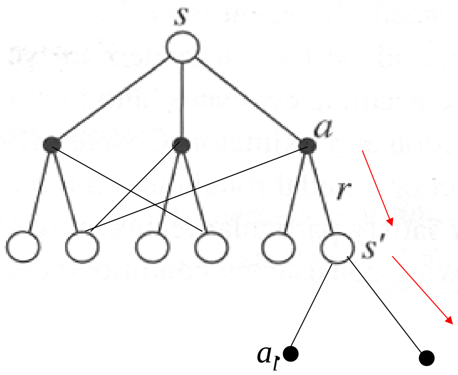
Equazioni di ottimalità di Bellman

V*(s) di uno stato, quando viene scelta la policy ottima, deve essere uguale al valore atteso del reward per l'azione migliore per lo stato s.

$$V^*(s) = \max_{a_j} \sum_{s'} P_{s \to s'|a_j} \left[R_{s \to s'|a_j} + \gamma V^*(s') \right]$$

$$Q^*(s, a_j) = \sum_{s'} P_{s \to s'|a_j} \left[R_{s \to s'|a_j} + \gamma \max_{a'} Q^*(s', a') \right]$$

Calcolo ricorsivo della value function Q



$$Q^\pi(s,a) = E_\pi\{R_t \mid s_t = s,\ a_t = a\} = E_\pi\left\{ \sum_{k=0}^{\infty} \gamma^k r_{t+k+1} \middle| s_t = s, a_t = a \right\}$$

$$\boxed{= \sum_{s'} P_{s \to s'|a} \left[R_{s \to s'|a} + \gamma \sum_{l} \pi(s', a_l) Q^\pi(s', a_l) \right]}$$

Q Functions

$$\pi^*(s) = \arg\max_a \sum_{s'} P^a_{s->s'}\left[R^a_{s->s'} + \gamma V^\pi(s')\right] = \arg\max_a Q(s,a)$$

V = Cumulative reward of being in *s* and choosing a_j. $Q^\pi(s,a_j) = \sum_{s'} P_{s\rightarrow s'|a_j}\left[R_{s\rightarrow s'|a_j} + \gamma V^\pi(s')\right]$

Idea chiave:

- Unire il rinforzo che si ottiene passando da uno stato al successivo in un'unica funzione

$$Q(s,a) = \left[R^a_{s->s'} + \gamma V^\pi(s')\right]$$

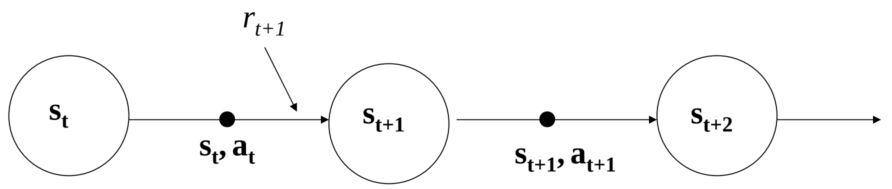
- Questa funzione valuta la bontà dell'azione e non più dello stato ($a = \pi(s)$).

- A questo punto posso massimizzare Q senza conoscere separatamente il reward istantaneo e la value function come:

$$\pi^*(s) = \text{argmax}_a\, Q(s,a)$$

Q = Cumulative reward of being in *s* and taking action *a*.

Rappresentazione grafica



V(s_t) V(s_{t+1})

One step for Value Iteration

Q(s_t, a_t) Q(s_{t+1}, a_{t+1})

One step for **Q Iteration**

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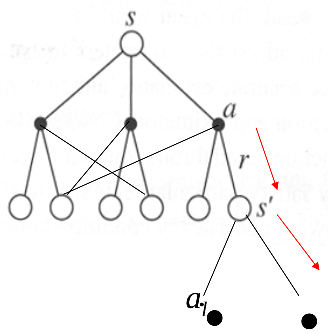
Come apprendere Q: SARSA

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha\left[r_{t+1} + \gamma Q\left(s_{t+1}, a_{t+1}\right) - Q\left(s_t, a_t\right)\right]$$

1) Apprendiamo il valore di Q per una policy data (on-policy).

2) Dopo avere appreso la funzione Q, possiamo modificare la policy in modo da migliorarla.



SARSA Algorithm (progetto)

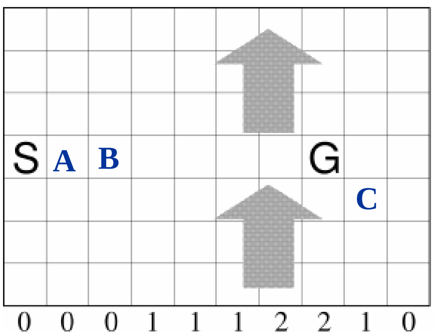
```
Q(s,a) = rand();   // ∀s, ∀a, eventualmente Q(s,a) = 0
Repeat                          // for each episode
{         s = s0;
          Repeat                // for each step of the single episode
          {
                    a = Policy(s);          // ε-greedy??
                    s_next = NextState(s,a);
                    reward = Reward(s,s_next,a);
                    a_next = Policy(s_next);       // ε-greedy?
                    Q(s,a) = Q(s,a) + α [reward + γ Q(s_next, a_next) – Q(s,a)];
                    s = s_next;
          }                     // until last state
}                               // until the end of learning
```

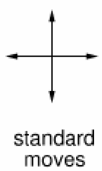
1) Apprendiamo il valore di Q per una policy data (on-policy).
2) Dopo avere appreso la funzione Q, possiamo modificare la policy in modo da migliorarla.

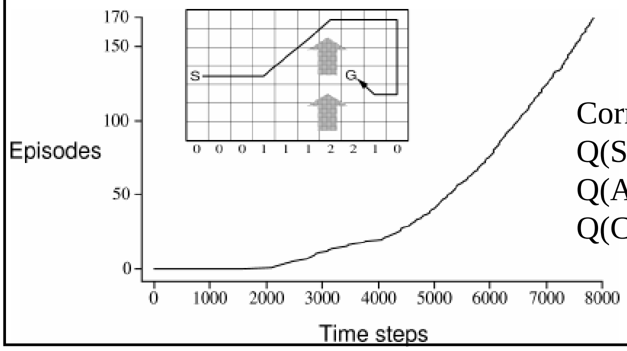
Come integrare i due passi?

Esempio

Q(s,a) iniziale = 0.01
r = 0 se s' = G,
altrimenti r = -1.
π(s,a) data.







Correzione di Q ad un passo:
Q(S, east) = -0.09
Q(A, east) = -0.09
Q(C, west) = 0

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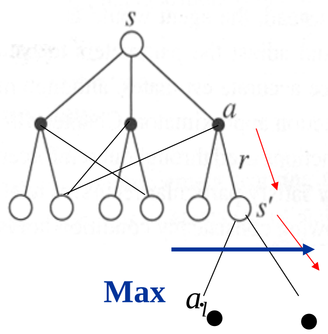
Off-policy Temporal Difference: Q-learning

$$\boxed{Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha\left[r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1},a_{t+1}) - Q(s_t,a_t)\right]}$$

Non imparo semplicemente la funzione valore Q, ma la funzione valore Q ottima.

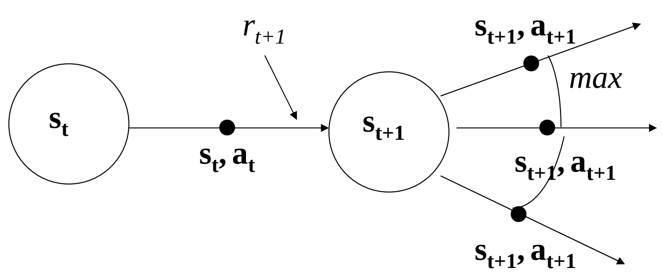
In s, scelgo un ramo del grafo, e poi **decido** ad un passo come continuare.



Q-learning algorithm (progetto)

```
Q(s,a) = rand();        // ∀s, ∀a, Q(s,a) = 0 evetualmente
Repeat                          // for each episode
{         s = s0;
          Repeat                // for each step of the single episode
          {
                    a = Policy(s);        // eventualmente ε-greedy
                    s_next = NextState(s,a);
                    reward = Reward(s,s_next,a);
                    a_next = Policy(s_next);        // eventualmente ε-greedy
                    Q(s,a) = Q(s,a) + α [reward + γ max Q(s_next, a_next) – Q(s,a)];
                    s = s_next;
                    UpdatePolicy(a_next, s_next);
          }             // until last state
}                       // until the end of learning
```


Rappresentazione grafica



$Q(s_t, a_t)$ $Q(s_{t+1}, a_{t+1})$

One step for **Q Iteration**

Viene migliorata la policy al tempo t+1.

Sommario

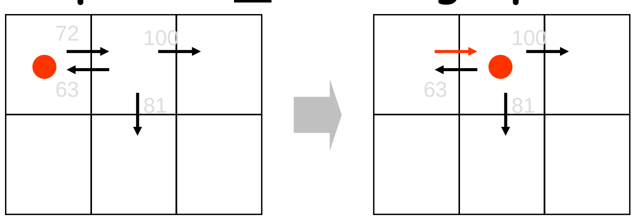
Dal modello dell'ambiente all'esperienza

Q-learning

Esempio

Example 1 – *Q* Learning Update

- $\gamma = 0.9$



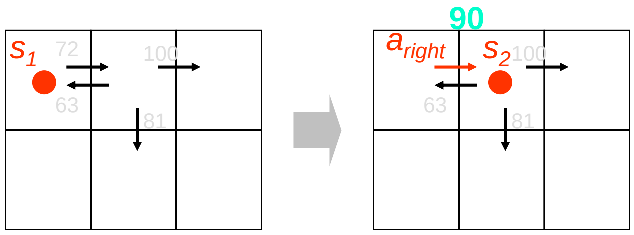
0 reward received

Esempio tratto dai lucidi del corso di Brian C. Williams su RL.
Modificati dalle slide di: Manuela Veloso, Reid Simmons, & Tom Mitchell, CMU

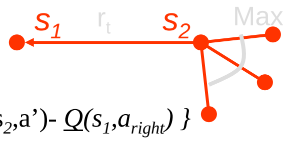
Apprendimento della funzione valore Q. Versione SARSA.

Example 1 – *Q* Learning Update

- $\gamma = 0.9$
- $\alpha = 0.1$



0 reward received in the transition



$Q(s_1,a_{right}) \leftarrow Q(s_1,a_{right}) + \alpha\{\, r(s_1,a_{right}, s_2) + \gamma \max_{a'} Q(s_2,a') - Q(s_1,a_{right}) \,\}$

$\leftarrow 72 + \alpha\, [0 + 0.9 \max\{63, 81, 100\} - Q(s_1,a_{right})\,]$

$\leftarrow 72 + \alpha(90 - 72) = 1.8$

In grigio i valori di Q(s,a). Nessun reward istantaneo.
Correzione di $Q(s_1,a_{right})$
La correzione va a 0 quando $Q(s_1,a_{right}) = 90$

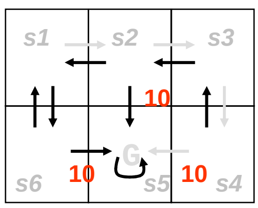
Example 2: Q-Learning Iterations: Episodic

- Start at upper left; Selected policy: move clockwise; Table initially 0; $\gamma = 0.8$.

Possibili transizione sono segnate con frecce nere e grigie.

$\alpha = 1$

Reward istanteo in rosso



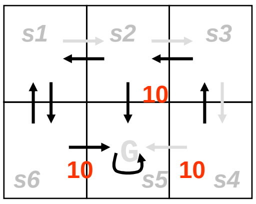
$$Q(s_t,a_t) \leftarrow \left[r_{t+1} + \gamma \max_{a_{t+1}} Q(s_{t+1},a_{t+1}) \right]$$

Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0			

Q-Learning Iterations

- Start at upper left – move clockwise; table initially 0; γ = 0.8

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

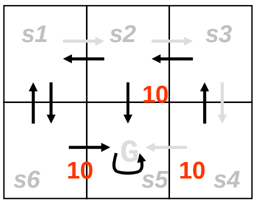


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	

Q-Learning Iterations

- Start at upper left – move clockwise; γ = 0.8

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

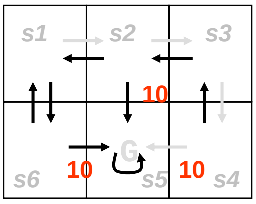


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{Q(s5a)\} =$ $10 + 0.8 \times 0 = \mathbf{10}$

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

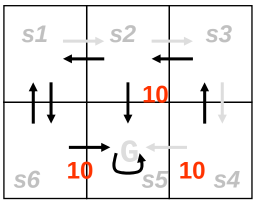


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{Q(s5,a)\}$= 10 + 0.8 x 0 = 10
0	0	$r + \gamma \max_{a'} \{Q(s4,W), Q(s4,N)\}$ = 0 + 0.8 x max{10,0} = 8	

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

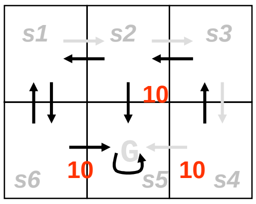


Q(S1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{Q(s5,loop)\}$= 10 + 0.8 x 0 = 10
0	0	$r + \gamma \max_{a'} \{Q(s4,W), Q(s4,N)\}$ = 0 + 0.8 x max{10,0} = 8	10
0	$r + \gamma \max_{a'} \{Q(s3,W), Q(s3,S)\}$ = 0 + 0.8 x max{0,8} = 6.4		

Q-Learning Iterations

- Start at upper left – move clockwise; $\gamma = 0.8$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$

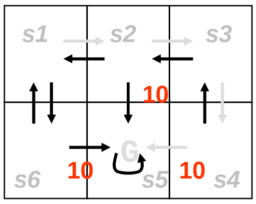


Q(s1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	$r + \gamma \max_{a'} \{Q(s5,loop)\} = 10 + 0.8 \times 0 = 10$
0	0	$r + \gamma \max_{a'} \{Q(s4,W), Q(s4,N)\} = 0 + 0.8 \times \max\{10,0\} = 8$	10
0	$r + \gamma \max_{a'} \{Q(s3,W), Q(s3,S)\} = 0 + 0.8 \times \max\{0,8\} = 6.4$	8	10

Q-Learning Iterations: improving policy

- Start at upper left – move clockwise; $\gamma = 0.8$

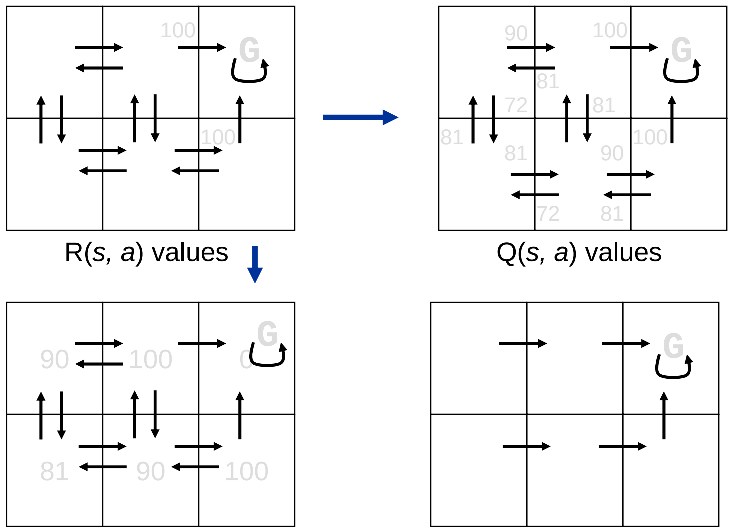
$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s',a')$$



Calcolo $Q(s_2, S) = r + \gamma \max_{a'} \{Q(s5,loop)\} = 10 + 0.8 \times 0 = \mathbf{10}$

Ricalcolo $Q(s1, E) = r + \gamma \max_{a'} \{Q(s2,E), Q(s2,W), Q(s2, S)\} = r + \gamma \max_{a'} \{6.4, 0.0, 10.0\}$ ➔ **South = $\pi(s_2)$!**

Example 3
Comparison functions V and Q



R(*s*, *a*) values

Q(*s*, *a*) values

V*(*s*) values

One Optimal Policy

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